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PROPAGATION OF STRESS WAVES IN LAYERED MEDIA UNDER IMPACT LOADING (ACOUSTICAL APPROXIMATION)

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UDC 534.211:539.42:539.63

1. INTRODUCTION

The impact loading of various bodies and structures by the detonation of an attached high-explosive charge, the firing of a projectile (driver), or thermal irradiation with a pulse of duration $\sim 10^{-9}$ sec can result in scabbing of the loaded bodies near their free surfaces, which originates in the unloading phase under the action of a stress wave. The action of tensile stress can be abated and the danger of scabbing can be diminished by the application of special layered systems, in which the generated shock impulse is partitioned at the layer interfaces into a branched system of compression and tension waves. It is technologically feasible at the present time to construct layered systems and structures from various types of materials by, e.g., the explosive welding of metal layers not amenable to conventional welding techniques, vacuum evaporation or detonation flame spraying of condensed films, the bonding of a series of layers, etc. The problems of shock transmission in layered systems have been investigated in studies of the influence of the parameters of colliding plates and buffer layers on the scabbing process [1] and on the quality of a welded joint between bonded materials in explosive welding [2]. An analysis of the wave processes for two- and three-layer systems has been carried out in [3-5]. A detailed theoretical and experimental study of the attenuation of shock waves in layered materials is given in [6]. The propagation of acoustic and electromagnetic waves in layered media has also been investigated in application to geophysical problems [7].

The objective of the present study is to analyze the generation of stress waves in a planar layered medium under impact loading in the acoustical approximation and to explore the possibility of preventing scabbing.

2. MODEL OF AN ELASTIC LAYERED MEDIUM

Let a layered medium consist of n different layers. A schematic diagram of such a medium of length L in the one-dimensional planar case is shown in Fig. 1. Each i-th layer of the meidum (i = 1, 2,...,n) is characterized by the true density ρ_i^0, the dynamic rigidity $Z_i = \rho_i^0 a_i$ (a_i is the longitudinal sound velocity in the i-th layer), and the length l_i $(L = \Sigma_{i=1}^n l_i)$. The quantity Z_i is also called the acoustic impedance and is related as follows to the elastic modulus of the material E_i $(a_i = \sqrt{E_i/\rho_i^0})$: $Z_i = \sqrt{E_i \rho_i^0}$. We denote the boundary between the i-th and (i + 1)-st layer by K_i. We assume that the impact loads are not too strong, so that the problem can be restricted to the acoustical approximation, i.e., we assume that the rigidity of the layers Z_i does not depend on the intensity of the transmitted waves and, hence, that the conditions $\rho_i^0 = \rho_{i0}^0$, $a_i = a_{i0}$ hold everywhere; then $Z_i = Z_{i0}$ (ρ_{i0}^0, a_{i0} correspond to the standard initial conditions $p_0 = 0$, $T_0 = 300°K$). We assume that a rectangular compression pulse J_1^I of duration t^w is generated in the first layer as a result of impact action along the r axis. In the subsequent transmission of J_1^I through the layers, multiple reflections take place at the boundaries K_i owing to the differences in the rigidities Z_i of the layers; these reflections produce reflected pulses J_i^R and transmitted pulses J_{i+1}^T of the same duration t^w. The pulses J_i^R and J_{i+1}^T can be either compression or tension pulses, depending on the ratio between the rigidities Z_i and Z_{i+1}. If the tensile stresses in the i-th layer exceed a certain threshold value σ_i^* characterizing the strength properties of the material of the i-th layer in tension under dynamic loading, scabbing will be possible inside the layer either almost instantaneously [8] or with a certain delay [9]. We assume that the tensile strength in each intermediate layer K_i, denoted by $\sigma_{i,i+1}^*$, is large and at

Ufa. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 125-133, January-February, 1985. Original article submitted December 7, 1983.

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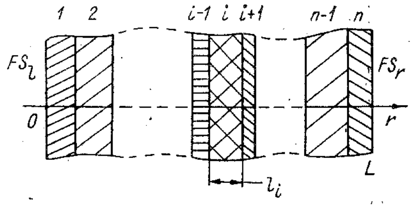

Fig. 1

least as great as the strengths σ_i^*, σ_{i+1}^* of the layers adjacent to K_i. In the case $\sigma_{i,i+1}^* < \sigma_i^*$, σ_{i+1}^*, destruction will take place along the bonding plane of the layers K_i.

We consider the process of trasnmission of a rectangular pulse in a layered medium (see Fig. 1). At the instant of transmission of the stress pulse J_i^I across the boundary K_i separating the layers, the conditions of continuity of the stress σ and displacements u on the left and right of the boundary K_i must be satisfied:

$$\sigma_i = \sigma_{i+1},\ u_i = u_{i+1}\ (\text{for}\ i = 1, 2, \ldots, n-1). \tag{2.1}$$

Separating out the incident (J_i^I), reflected (J_i^R), and transmitted (J_{i+1}^T) pulses at the boundary K_i, we can write the interface conditions (2.1) in the form [10]

$$\sigma_i^I + \sigma_i^R = \sigma_{i+1}^T,\ \ u_i^I + u_i^R = u_{i+1}^T. \tag{2.2}$$

Let the displacement u and, hence, the velocity v of particles of the medium in the i-th layer be described by a certain function φ of the coordinates r and t:

$$u_i = \varphi(r \pm a_i t),\ v_i = \dot{u}_i = \pm a_i \varphi'(r \pm a_i t), \tag{2.3}$$

which satisfies the equation of motion

$$\rho_i^0 \partial^2 u_i / \partial r^2 = \partial \sigma_i / \partial r. \tag{2.4}$$

Bearing Hooke's law ($\sigma_i = E_i \partial u_i / \partial r$) in mind, we obtain

$$\sigma_i = E_i \varphi'(r \pm a_i t) = \rho_i^0 a_i v_i = Z_i v_i. \tag{2.5}$$

After differentiating the second relation in (2.2) with allowance for (2.5) and the direction of motion of the reflected and transmitted pulses, we have

$$(-\sigma_i^I + \sigma_i^R)/Z_i = \sigma_{i+1}^T / Z_{i+1}. \tag{2.6}$$

From the system (2.2) and (2.6) we obtain at once

$$\sigma_i^R = \sigma_i^I (Z_{i+1} - Z_i)/(Z_i + Z_{i+1}),\ \ \sigma_{i+1}^T = \sigma_i^I 2Z_{i+1}/(Z_i + Z_{i+1}). \tag{2.7}$$

The relation for the velocities is found analogously:

$$v_i^R = v_i^I (Z_i - Z_{i+1})/(Z_i + Z_{i+1}),\ \ v_{i+1}^T = v_i^I 2Z_i/(Z_i + Z_{i+1}). \tag{2.8}$$

An analysis of (2.7) and (2.8) leads to the following conclusion. Depending on the ratios of the rigidities Z_i and Z_{i+1}, the transmission of the pulse J_i^I will either be partially impeded during transition into a less rigid medium with an increase in the particle velocity v_i (in this case $\operatorname{sgn}\sigma_i^R = -\operatorname{sgn}\sigma_i^I$, and $|\sigma_{i+1}^T| < |\sigma_i^I|$, $v_{i+1}^T > v_i^I$) or be amplified with a decrease in the velocity v_i during transition into a more rigid medium (in which case $\operatorname{sgn}\sigma_i^R = \operatorname{sgn}\sigma_i^I$ and $|\sigma_{i+1}^T| > |\sigma_i^I|$, $v_{i+1}^T < v_i^I$). The effect of amplification of the bulk particle velocity when a pulse exits into a less rigid medium has been utilized in [11] to impart large velocities to a driver plate by the application of a buffer layer with a lower rigidity. If the rigidities in the adjacent layers coincide, the pulse J_i^I will cross the boundary K_i without reflection:

$$\sigma_i^R = 0,\ \ \sigma_{i+1}^T = \sigma_i^I,\ \ v_i^R = 0,\ \ v_{i+1}^T = v_i^I. \tag{2.9}$$

When the interface K_i represents a free surface FS (see the surface $K_n = FS_r$ in Fig. 1),

$$\sigma_n^R = -\sigma_n^I,\ \ \sigma_{r=L}^T = 0,\ \ v_n^R = v_n^I,\ \ v_{r=L}^T = 2v_n^I. \tag{2.10}$$

If the (i + 1)-st layer represents an absolutely rigid medium,

$$\sigma_n^R = \sigma_n^I,\ \ \sigma_{r=L}^T = 2\sigma_n^I,\ \ v_n^R = -v_n^I,\ \ v_{r=L}^T = 0. \tag{2.11}$$

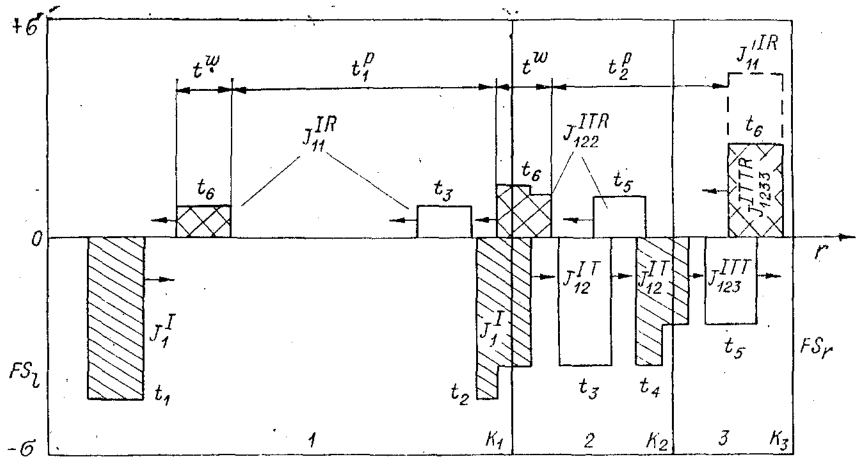

Fig. 2

Relations (2.10) and (2.11) follow from (2.7) and (2.8), in which σ^T and v^T correspond to interaction of the pulse with the boundary K_r at $r = L$ during the time t^w.

Equations (2.4) and (2.5) together with the appropriate initial and boundary conditions completely determine the motion of the pulse J_1^I in the layered medium. If the parameters of the pulse J_1^I are known from the boundary conditions of the problem, relations (2.7) and (2.8) can be used to find the stresses and particle velocities in the layers. Below, we discuss several reasonably simple problems, the solutions of which can be used to exhibit the fundamental laws inherent in the generation and interaction of stress waves in layered media and to draw a number of conclusions applicable to more complex situations. These problems have been chosen to illustrate the proposed computational procedure, which makes it possible to solve not only direct problems, but also inverse problems in the design of layered blast shields [1, 12] (to determine the rigidities Z_i and thickness l_i of the layers).

3. ANALYSIS OF WAVE PROCESSES IN THE THREE-LAYER PLATE

The loading of a homogeneous plate with a compression pulse J^I of duration t^w, where the rigidity of the plate is uniform and equal to Z over the entire length (total thickness) L, has the effect of producing a tension pulse J^R of amplitude $\sigma^R = -\sigma^I$ near the backside free surface FS_r. If a multilayer plate of the same length $L = \Sigma_{i=1}^n l_i$ is used instead of a homogeneous plate, where the rigidity $Z_1 = Z$ and the subsequent layers have decreasing Z_i ($Z_i > Z_{i+1}$ for $i = 1, 2, \ldots, n - 1$), then the tension pulse generated near FS_r will have a smaller amplitude, because the transmitted compression pulse will be partially impeded at each boundary K_i. The amplitude of the reflected tension pulse σ_i^R at the boundary K_i, according to (2.7), can be controlled by selection of the layer rigidities Z_i. In order for the reflected tension pulses J_i^R moving toward the free surface $K_0 = FS_l$ not to overtake one another (resulting in summation of the pulses J_i^R and amplification of the resultant pulse amplitude) and to be separated by a certain delay time t_i^p, it is necessary to choose sufficient thickness l_i of the layers on the basis of the condition $2\Delta t_{i+1} \geqslant t^w (t^w + t_i^p = 2\Delta t_{i+1}, \Delta t_{i+1} = l_{i+1}/a_{i+1})$. If we set $\Delta t_{i+1} = t^w/2$, the total thickness L of the plate will be a minimum in this case, and the tension pulses J_i^R will travel in tight proximity to one another ($t_i^p = 0$). We consider in detail the process of the inception and evolution of the system of incident, reflected, and transmitted pulses in a three-layer plate.

Problem 1. Let a rectangular compression pulse J_1^I be generated as a result of impact in the first layer of a three-layer plate of thickness $L = \Sigma_{i=1}^3 l_i$ ($i = 1, 2, 3$) with specified layer thicknesses meeting the condition $Z_1 > Z_2 > Z_3$ (see time t_1 in Fig. 2), and let the condition $l_i^w < l_i$ be satisfied by the widths $l_i^w = a_i t^w$ of the incident pulse J^I (of duration t^w) in the layers l_i of the plate. We seek the stresses of the pulses J_i^R and J_{i+1}^T as a function of the rigidities Z_i.

We introduce the following subscript and superscript indexing system for the pulses generated in the layered medium. A digit subscript indexes the order of the layer in which the pulse is acting at a given time, and a letter superscript (I, R, T) indicates the way in which the pulse is formed. For example, J_{123}^{ITT} is the pulse transmitted into the third layer, J_{122}^{ITR} is the pulse reflected from the boundary K_2 and acting in the second layer, and J_{1233}^{ITTR} is

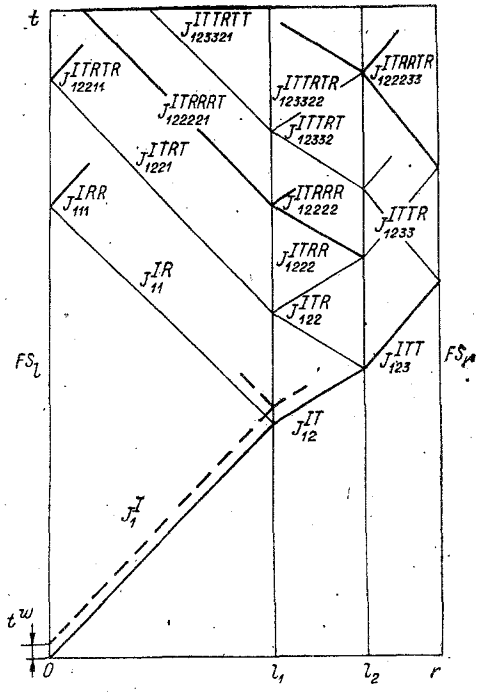

Fig. 3

the pulse generated in the reflection of J^{ITT}_{123} from the free surface K_3 at the right end. Every addition of a superscript R indicates a reversal of the direction of motion of the pulse, and the number of R's indicates the degree of branching of the pulse (secondary, tertiary, etc.), so that J^{ITTRR}_{12333} denotes a secondary pulse moving in the third layer from K_2 toward K_3. The sign of the stresses $\sigma^{I\ldots Q}_{1\ldots i}$ (compression or tension) of the pulse $J^{I\ldots Q}_{1\ldots i}$ is determined by relation (2.7).

Figure 2 shows a different times the process of transmission of the pulse J^{I}_{1} across the boundaries K_1, K_2, and K_3 of a three-layer plate with the formation at K_1 and K_2 of unloading waves (traveling from right to left), which initially relieve the compression region and are then transformed into tension pulses (see the pulses J^{IR}_{11} and J^{ITR}_{122} at times t_2 and t_3). A tension pulse J^{ITTR}_{1233} is formed at K_3. In connection with the motion of the pulses through the layers of the plate it is assumed for convenience that $a_1 = a_2 = a_3$ and that the reduction in the rigidities is determined by the densities of the layers, $\rho^0_1 > \rho^0_2 > \rho^0_3$. The system of tension pulses J^{IR}_{11}, J^{ITR}_{122}, and J^{ITTR}_{1233} acting in the plate at time t_6 is distinguished by identical cross-hatching in Fig. 2. For comparison, the pulse J'^{IR}_{11} that would have occurred if the plate plate were homogeneous over its entire thickness with $Z = Z_1$ is superimposed on the pulse J^{ITTR}_{1233} in the third layer near the surface K_3. In layers 1 and 2, as in layer 3, the amplitudes of J^{IR}_{11} and J^{ITR}_{123} are smaller than the amplitude of J'^{IR}_{11}. It is clear that the amplitude $\sigma^{I\ldots Q}_{1\ldots i}$ of the tension pulse acting in the i-th layer can be made smaller than the threshold level of the fracture stresses σ^{*}_{i}.

It would seem that the problem is solved. However, if we trace the subsequent evolution of a tension pulse, say J^{ITR}_{122}, as it is transmitted across K_1 into a more rigid medium, moving from right to left in the direction of K_0 (the process of transmission of J^{ITR}_{122} from layer 2 into layer 1 is shown at time t_6 in Fig. 2), we find that the amplitude of the tension pulse J^{ITRT}_{1221} has increased. The secondary pulse J^{ITRR}_{1222} reflected from K_1 into layer 2 is a tension pulse. Then the pulse J^{ITRR}_{1222} exits across K_2 into the less rigid layer 3, in which case the amplitude of the transmitted tension pulse J^{ITRRT}_{12223} decreases, and the pulse J^{ITRRR}_{12222} reflected from K_2 is a compression pulse. All secondary, tertiary, etc., tension and compression pulses generated in transition across the interfaces K_i have small amplitudes (corresponding estimates will be given below) and, by interfering with the primary tension pulses shown in Fig. 2 at time t_6, are capable of amplifying or attenuating their action. These pulses are disregarded from now on, since their contribution is small. The complete system of transmitted and reflected compression and tension pulses is shown in the r–t diagram of Fig. 3. The lines of the main contour correspond to compression, and those of the auxiliary contour represent tension. The solid lines indicate the motion of the leading edge of all the stress pulses,

TABLE 1

n	$\sigma_{12\ldots n}^{IT\ldots T}$	$\sigma_{12\ldots nn(1-n)\ldots 1}^{IT\ldots TRT\ldots T}$	n	$\sigma_{12\ldots n}^{IT\ldots T}$	$\sigma_{12\ldots nn(n-1)\ldots 1}^{IT\ldots TRT\ldots T}$
1	1	1	9	0,300	—0,807
2	0,777	—0,888	10	0,283	—0,805
3	0,533	—0,853	20	0,200	—0,795
4	0,457	—0,835	30	0,162	—0,792
5	0,406	—0,825	40	0,140	—0,790
6	0,369	—0,818	50	0,125	—0,789
7	0,340	—0,814	60	0,114	—0,788
8	0,318	—0,810			

TABLE 2

n	$\sigma_{12\ldots n}^{IT\ldots T}$	$\sigma_{12\ldots nn(n-1)\ldots 1}^{IT\ldots TRT\ldots T}$
1	1	1
2	0,666	—0,888
3	0,250	—0,563
4	0,063	—0,262
5	0,012	—0,095
6	0,001	—0,028

and the dashed line represents the motion of the trailing edge of only the pulses J_1^I, J_{12}^{IT}, and J_{11}^{IR}. In Fig. 3 the rigidities Z_i satisfy the condition $Z_1 > Z_2 > Z_3$, and the velocities $a_2 > a_1 > a_3$. It is evident from the r–t diagram that tension and compression pulses moving in opposite directions occur in the layered medium, depending on the ratio of the rigidities Z_i/Z_{i+1}. We call attention to the following. The compression pulse J_{111}^{IRR}, beginning with time t_a, and the compression pulse J_{12211}^{ITRTR}, beginning with time t_b, will attenuate the tension pulses traveling oppositely to them. The subsequent behavior of these pulses is similar to that of the pulse J_1^I, i.e., the compression pulses are attenuated at the boundaries K_1 and K_2 and, in turn, generate tension pulses in the direction of K_0. The times t_a and t_b can be controlled by decreasing the thickness l_1. This consideration leads to the formulation of the problem of how to choose the thickness l_i so that the secondary compression waves will attenuate the action of the tensile stresses at the most dangerous location, e.g., in a particular individual layer or at the interface between two layers.

We now return to the primary tension pulses. The pulses J_{1233}^{ITTR} and J_{122}^{ITR} are amplified in transition to more rigid layers (see the data of Figs. 2 and 3), and the situation is entirely realistic, in which the level of the tensile stresses in the i-th layer can exceed the fracture threshold σ_i^*. The greatest amplification in the given problem is exhibited by the tension pulse J_{1233}^{ITTR} transmitted in succession from the least rigid third layer into the most rigid first layer. Let us estimate the amplitude of the pulse J_{123321}^{ITTRTT} transmitted into the first layer. Using relation (2.7), we can show that the maximum attenuation of the compression pulse J_1^I is attained in the last layer, where the amplitude is given as

$$\sigma_{123}^{ITT} = \sigma_1^I 2^2 Z_2 Z_3/[(Z_1+Z_2)(Z_2+Z_3)]. \tag{3.1}$$

The amplitude of the tension pulse generated in reflection of the compression pulse J_{123}^{ITT} from the free surface K_3 and returned to the first layer is determined from (2.7) with allowance for (3.1) and is equal to

$$\sigma_{123321}^{ITTRTT} = -\sigma_1^I 2^4 Z_1 Z_2^2 Z_3/[(Z_1+Z_2)^2(Z_2+Z_3)^2]. \tag{3.2}$$

If we assume that the decrement of the rigidities ΔZ_i is identical at both interfaces K_i, i.e., $\Delta Z_i = Z_1/3$, it follows at once from (3.1) and (3.2) that $\sigma_{123}^{ITT} = 0.533\sigma_1^I$, $\sigma_{123321}^{ITTRTT} = -0.853\sigma_1^I$, i.e., the tension pulse is attenuated ~15% in the first layer relative to σ_1^I (see Figs. 2 and 3). The attenuation of the tension pulse here is not caused by dissipative processes occurring in the layered medium during transmission of the shock pulse, but is determined only by the corresponding choice of geometrical dimension (l_i) and material properties (ρ_i^0, a_i) of the layers comprised in the plate. In this plan, an attenuation of the tensile stresses by even 10-15% is quite appreciable.

Relations (3.1) and (3.2) are readily generalized to an n-layer plate. For the pulse transmitted from the first into the j-th layer ($1 \leqslant j \leqslant n$) and then returned (by reflection from K_j) into the i-th layer ($1 \leqslant i \leqslant j \leqslant n$) we have

$$\begin{gathered} \sigma_{12\ldots j}^{IT\ldots T} = \sigma_1^I 2^{(j-1)} Z_2 Z_3 \ldots Z_j/[(Z_1+Z_2)(Z_2+Z_3)\ldots(Z_{j-1}+Z_j)], \\ \sigma_{12\ldots jj(j-1)\ldots i}^{IT\ldots TRT\ldots T} = \sigma_1^I \frac{2^{(2j-i-1)} Z_2 Z_3 \ldots Z_{i-1} Z_i^2 Z_{i+1}^2 \ldots Z_{j-1}^2 Z_j (Z_{j+1}-Z_j)}{(Z_1+Z_2)\ldots(Z_{i-1}+Z_i)(Z_i+Z_{i+1})^2\ldots(Z_{j-1}+Z_j)^2(Z_{j+1}+Z_j)}. \end{gathered} \tag{3.3}$$

From (3.3) for j = n and i = 1 we obtain

$$\sigma_{12\ldots n}^{IT\ldots T} = \sigma_1^I 2^{(n-1)} Z_1 Z_2 \ldots Z_n/[(Z_1+Z_2)(Z_2+Z_3)\ldots(Z_{n-1}+Z_n)],$$

TABLE 3

Three-layer plate	1: $Z_2=0,666Z_1$, $Z_3=0,333Z_1$	2: $Z_2=0,5Z_1$, $Z_3=0,25Z_1$	3: $Z_2=0,354Z_1$, $Z_3=0,084Z_3$	Degree of branching and type of stress pulses
σ_1^I	1	1	1	Primary pulses compression
σ_{12}^{IT}	0,800	0,500	0,523	
σ_{123}^{ITT}	0,533	0,250	0,199	
σ_{11}^{IR}	—0,200	—0,500	—0,477	Primary pulses tension
σ_{122}^{ITR}	—0,266	—0,250	—0,323	
σ_{1221}^{ITRT}	—0,320	—0,375	—0,477	
σ_{1233}^{ITTR}	—0,533	—0,250	—0,199	
σ_{12332}^{ITTRT}	—0,711	—0,375	—0,322	
σ_{123321}^{ITTRTT}	—0,853	—0,562	—0,477	
σ_{111}^{IRR}	0,200	0,500	0,477	Secondary pulses compression
σ_{12211}^{ITRTR}	0,320	0,375	0,477	
$\sigma_{1233211}^{ITTRTTR}$	0,853	0,562	0,477	
σ_{1222}^{ITRR}	—0,053	—0,125	—0,154	Secondary pulses tension
σ_{12223}^{ITRRT}	—0,035	—0,062	—0,059	
σ_{12333}^{ITTRR}	—0,177	—0,125	—0,123	
σ_{123322}^{ITTRTR}	—0,142	—0,187	—0,154	
σ_{12222}^{ITRRR}	0,017	0,062	0,095	Tertiary pulses compression
σ_{122221}^{ITRRRT}	0,021	0,093	0,141	

$$\sigma_{12\ldots nn(n-1)\ldots 1}^{IT\ldots TRT\ldots T} = -\sigma_1^I 2^{2(n-1)} Z_1 Z_2^2 \ldots Z_{n-1}^2 Z_n / \left[(Z_1+Z_2)^2 \ldots (Z_{n-1}+Z_n)^2\right]. \tag{3.4}$$

To what extent is attenuation possible for the tension pulse $J_{12\ldots nn(n-1)\ldots 1}^{IT\ldots TRT\ldots T}$ occurring in a multilayer plate in comparison with the tension pulse J_{11}^{IR} in a homogeneous plate? To answer this question we examine two special cases.

1. Let the change of the rigidities Z_i in each layer of the plate be constant and let it obey the diminishing arithmetic progression law $\Delta Z_i = \Delta Z = Z_1/n$; also, $Z_{i+1} = Z_1 - i\Delta Z = Z_1(n-i)/n$, whereupon it follows from (3.4) that

$$\sigma_{12\ldots n}^{IT\ldots T} = \sigma_1^I 2^{n-1}(n-1)!/[(2n-1)!!], \qquad \sigma_{12\ldots nn(n-1)\ldots 1}^{IT\ldots TRT\ldots T} = -\sigma_1^I 2^{2(n-1)} n!\,(n-1)!/[(2n-1)!!]^2. \tag{3.5}$$

The values of the stress attenuation coefficients relative to σ_1^I, calculated according to (3.5), are shown in Table 1 for equal values of n. For large n, we obtain the following from (1.5) with the application of Stirling's formula:

$$\sigma_{12\ldots n}^{IT\ldots T} \approx 0.5\sigma_1^I \sqrt{\pi/(n-1)}, \quad \sigma_{12\ldots nn(n-1)\ldots 1}^{IT\ldots TRT\ldots T} \approx -\sigma_1^I \pi n/4(n-1). \tag{3.6}$$

It follows from (3.6) that the amplitude of the compression pulse in a multilayer plate can be reduced by a given factor, and the amplitude of the tension pulse returned to the first layer for sufficiently large n is approximately equal to $-0.25\pi\sigma_1^I$ (see the data of Table 1 for n = 30, 40, 50, and 60).

2. In another special case, the rigidities Z_i of the successive layers decrease according to a geometric progression with the denominator $q = 1/n$, i.e., $Z_i = Z_1 q^{i-1}$. Then the following relations are valid:

$$\sigma_{12\ldots n}^{IT\ldots T} = \sigma_1^I [2/(n+1)]^{n-1}, \quad \sigma_{12\ldots nn(n-1)\ldots 1}^{IT\ldots TRT\ldots T} = -\sigma_1^I (n[2/(n+1)]^2)^{n-1}. \tag{3.7}$$

The values of the stress attenuation coefficients calculated according to (3.7) are shown in Table 2 for various n. Condition (3.7) is not readily satisfied, because, e.g., the last layer of a three-layer plate must have $Z_3 \precsim 10^{-1}Z_1$, and in a five-layer plate $Z_5 \precsim 10^{-3}Z_1$.

TABLE 4

σ	σ/σ_1^I	
	$Z_1>Z_2$	$Z_2=0,5Z_1$
σ_{12}^{IT}	$2Z_2/(Z_1+Z_2)$	0,666
σ_{11}^{IR}	$(Z_2-Z_1)/(Z_1+Z_2)$	−0,333
σ_{111}^{IRR}	$(Z_2-Z_1)/(Z_1+Z_2)$	0,333
σ_{122}^{ITR}	$-2Z_2/(Z_1+Z_2)$	−0,666
σ_{1221}^{ITRT}	$-4Z_1Z_2/(Z_1+Z_2)^2$	−0,888
σ_{1111}^{IRRR}	$-(Z_2-Z_1)^2/(Z_1+Z_2)^2$	−0,111
$\sigma_1^{\Sigma R}$	−1	−1
σ_{1222}^{ITRR}	$-2Z_2(Z_1-Z_2)/(Z_1+Z_2)^2$	−0,222
σ_{1112}^{IRRT}	$-2Z_2(Z_2-Z_1)/(Z_1+Z_2)^2$	0,222
$\sigma_2^{\Sigma R}$	0	0
$\sigma_{1,2}^{s}$	$2Z_2/(Z_1+Z_2)$	0,666
$\sigma_{1,2}^{R}$	$-2Z_2/(Z_1+Z_2)$	−0,666

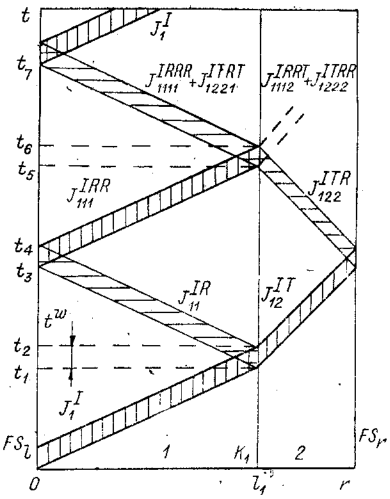

Fig. 4

Problem 2 (design problem). In a three-layer plate, let the primary tension pulses returned to the first layer in succession from K_1, K_2, and K_3 have identical amplitudes, i.e., let

$$\sigma_{11}^{IR}=\sigma_{1221}^{ITRT}=\sigma_{123321}^{ITTRTT}=\sigma^R. \tag{3.8}$$

What conditions must the layer rigidities Z_i satisfy in this case?

For i = 1 and j = 1, 2, 3 we obtain from (3.3)

$$\sigma_{11}^{IR}=\sigma_1^I(Z_2-Z_1)/(Z_1+Z_2),\quad \sigma_{1221}^{ITRT}=\sigma_1^I 2^2 Z_1Z_2(Z_3-Z_2)/[(Z_1+Z_2)^2(Z_2+Z_3)],$$
$$\sigma_{123321}^{ITTRTT}=-\sigma_1^I 2^4 Z_1Z_2^2Z_3/[(Z_1+Z_2)^2(Z_2+Z_3)^2]. \tag{3.9}$$

Solving (3.8) and (3.9) simultaneously for Z_2 and Z_3, we obtain $Z_2 = 0.354Z_1$, $Z_3 = 0.084Z_1$, and $\sigma^R = -0.477\sigma_1^I$, i.e., the amplitude of the tension pulses transmitted into the first layer is 47.7% in comparison with the amplitude of the incoming compression pulse. The following ratios of the rigidities are obtained in the solution of the analogous problem for a four-layer plate: $Z_2 = 0.426Z_1$, $Z_3 = 0.151Z_1$, $Z_4 = 0.036Z_1$, and $\sigma^R = -0.4\sigma_1^I$. This result shows that a predetermined condition $\sigma_1^R < \sigma_1^*$ can be satisfied and the danger of scabbing averted by the proper selection of the number of layers with specified properties in the compound plate. Table 3 gives the attenuation coefficients of the primary, secondary, and certain tertiary stress pulses for three-layer plates, in which the rigidities of the layers forming the plate vary according to: 1) a diminishing arithmetic progression law; 2) a diminishing geometric progression law; 3) the law derived in the analysis of problem 2. An analysis of the data in Table 3 shows that the most uniform and greatest attenuation of the primary tension pulses occurs for the third case. The secondary and tertiary tension pulses are small, and their amplitudes do not exceed 10–20% in comparison with σ_1^I.

4. TWO-LAYER PLATE

It was noted earlier that the generated secondary compression pulses can be made to attenuate the action of the secondary tension pulses in certain individual layers or at the layer junctions by proper selection of the layer thicknesses l_i in the impact loading of layered plates. We now consider the following problem.

Problem. Let a two-layer plate with $Z_1 > Z_2$ be impact-loaded, and let the layer thicknesses l_1 and l_2 be chosen so that the transit time of a stress pulse of duration t^w in the layers is identical, i.e., $\Delta t_1 = \Delta t_2$ ($\Delta t_i = l_i/a_i$, i = 1, 2). Moreover, let us assume for definiteness that $t^w < \Delta t_1$. What tensile stresses will be experienced by the layers of the plate and the interface K_1? The problem can be stated alternatively: Can l_1 and l_2 be chosen in such a way as to ensure the minimum possible level of tensile stresses at the boundary K_1?

An r–t diagram of the loading of such a two-layer plate is shown in Fig. 4. In transition across the interface the pulse J_1^I is partitioned into a compression pulse J_{12}^{IT} and a tension pulse J_{12}^{IR} (see times t_1 and t_2 in Fig. 4; compression and tension pulses are indicated by vertical and horizontal hatching, respectively). The compression pulse J_{12}^{IT} is converted in reflection from K_2 to an tension pulse J_{122}^{ITR} moving toward the surface K_1. The expansion pulse J_{11}^{IR} is converted in reflection from K_0 to a compression pulse J_{111}^{IRR}, which also moves toward K_1 (see times t_3 and t_4). The process of interaction of the pulses J_{111}^{IRR} and J_{122}^{ITR} with the boundary K_1 begins at time t_5 and terminates at t_6. The compression pulse J_{111}^{IRR} splits at K_1 into a transmitted compression pulse J_{1112}^{IRRT} and a reflected tension pulse J_{1111}^{IRRR}. The tension pulse J_{112}^{IRT}, which propagates into the first layer, is amplified, and is converted into a pulse J_{1221}^{ITRT}; the secondary tension pulse J_{1222}^{ITTT} is reflected in the second layer. Table 4 shows the coefficients of attenuation of the stresses in the pulses in terms of the rigidities Z_1 and Z_2, along with their numerical values for $Z_2 = 0.5Z_1$. We call attention to the following: For arbitrary values of Z_1 and Z_2 the sum of the secondary compression and tension pulses J_{1112}^{IRRT} and J_{1222}^{ITRR} traveling in the second layer is equal to zero (this sum pulse is represented by dashed lines in Fig. 4); on the other hand, the sum of the primary tension pulse J_{1221}^{ITRT} and the tertiary tension pulse J_{1111}^{IRRR} in the first layer is equal to a tension pulse with stress $\sigma_1^{\Sigma R} = -\sigma_1^I$, which begins to act at a time $t \geqslant t_5 + t^w/2$ (see Fig. 4). In subsequent reflection from the free surface K_0 the pulse $J_1^{\Sigma R}$ is converted to a compression pulse and, beginning at time t_7, the wave pattern becomes similar to that considered above. At the interface K_1 of the layers, compressive stresses with an amplitude $\sigma_{1,2}^S$ will act for a period t^w from time t_1 to t_2, and tensile stresses $\sigma_{1,2}^R$ will act from t_5 to t_6 at K_1 and in a tetragon next to K_1. In a different two-layer plate geometry where the time of transition of the compression pulse across K_1 did not coincide with the time of transition of the tension pulse J_{122}^{ITR}, the acting tensile stress at the interface K_1 would be $\sigma_{1,2}^R = \sigma_{1221}^{ITRT}$ (see Table 4). In contrast with a homogeneous plate with $Z = Z_1$, in which a tension pulse with $\sigma_{11}^{IR} = -\sigma_1^I$ is generated at the right free surface of the plate, in the two-layer plate a tension pulse with $\sigma_1^{\Sigma R}$ is generated in the first in the first layer next to the interface K_1, where the onset of scabbing is also possible. This important fact makes it possible to use blast shields not only on the backside of the loaded sample [13], but also on the impact side, in which case the frontal buffer layer must have a high rigidity. This conclusion is consistent with the data obtained in an investigation [1] of the influence of the placement of rigid and compliant mats on the destruction of a target. We also note here the very interesting experimental fact, presented and discussed in [14], that destruction takes place next to the interface for a container of two plates of identical material and equal thickness. The complete reproduction of the tension pulse with $\sigma_1^{\Sigma R} = -\sigma_1^I$ in the first layer is possible only for a two-layer plate; using relations (2.7)-(2.11) and the scheme of the formation of stress waves (see Fig. 3), we readily determine $\sigma_1^{\Sigma R}$ for a multilayer plate with $n > 2$.

The foregoing acoustical analysis is useful (because of its comparative simplicity) for the understanding and on-line prediction of the wave pattern generated in impact or explosive loading of multilayer plates. For a more complete description of the wave processes involved in layered media it is necessary to invoke elastoplastic models and to characterize the materials of the layers by equations of state that are valid over a wide range of shock wave intensities.

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ASYMPTOTIC ANALYSIS OF THE PLANE CONTACT PROBLEM OF ELASTICITY THEORY FOR A TWO-LAYER FOUNDATION

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An asymptotic analysis is presented of the plane contact problem of elasticity theory for a two-layer foundation that permits selection of some model of the upper relative to a thin layer (coating), depending on the relationship between the physicomechanical and geometric values of the coating and the support (elastic half plane).

1. Let us consider an elastic half plane ($y \leqslant 0$) with Poisson ratio ν_2 and shear modulus G_2. We assume that there is a relatively thin layer $0 \leqslant y \leqslant h(\nu_1, G_1)$ on and rigidly connected to the half-plane surface.* Let a rigid stamp, for which the shape of the foundation is described by a function f(x) even in x be impressed without friction by a force P on the upper boundary of such a composite medium. The boundary conditions of the problem posed are written in the form (the superscript 1 refers to the layer, and the superscript 2 to the half plane)

$$\begin{gathered} y=h:\ v^{(1)}=v_+(x)=-\delta+f(x),\quad \sigma_y^{(1)}=-\sigma_+(x)\quad (|x|\leqslant a),\\ \sigma_y^{(1)}=0,\ (|x|>a),\quad \tau_{xy}^{(1)}=\tau_+(x)=0\quad (|x|<\infty);\\ y=0:\ \sigma_y^{(1)}=\sigma_y^{(2)},\quad \tau_{xy}^{(1)}=\tau_{xy}^{(2)},\ v^{(1)}=v_-(x)=v^{(2)},\ u^{(1)}=u_-(x)=u^{(2)}. \end{gathered} \tag{1.1}$$

The stresses and strains vanish at infinity. Here δ is the rigid displacement of the stamp under the action of the force P applied thereto, $\sigma_\pm(x)$, $\tau_\pm(x)$ are the normal and tangential forces at the upper (plus sign) and lower (minus sign) faces of the layer, respectively, and $v_\pm$, $u_\pm$ are the vertical and horizontal displacements of the faces of the layer.

The formulated problem is reduced by integral transform methods [1] to the determination of the contact pressures $\sigma_+(x)$ from a convolution type integral equation of the first kind in a finite interval [2]

$$\int_{-a}^{a}\sigma_+(\xi)\,d\xi\int_{-\infty+ic}^{\infty+ic}\frac{L(\alpha)}{|\alpha|}\exp\left[-i\frac{\alpha}{h}(\xi-x)\right]d\alpha=2\pi\theta_1[\delta-f(x)]\quad(|x|\leqslant a); \tag{1.2}$$

$$L(u)=\frac{M+4|u|e^{-2|u|}-Ne^{-4|u|}}{M-(1+4u^2+NM)e^{-2|u|}+Ne^{-4|u|}}, \tag{1.3}$$

$$\mu_i=1-\nu_i^*,\ \varkappa_i=3-4\nu_i,\ \theta_i=G_i\mu_i^{-1}\ (i=1,2),\ n=\theta_1\theta_2^{-1},$$

$$M=(n\mu_1+\mu_2\varkappa_1)(n\mu_1-\mu_2)^{-1},\ N=(n\mu_1\varkappa_2-\mu_2\varkappa_1)(n\mu_1\varkappa_2+\mu_2)^{-1}.$$

Taking account of the notation

$$\begin{gathered} u=\alpha h,\ x=x'a,\ \xi=\xi'a,\ \lambda=ha^{-1},\\ \sigma_+(x)\,\theta_1^{-1}=q(x'),\ \delta=\Delta a,\ f(x)=r(x')\,a \end{gathered} \tag{1.4}$$

*We call a layer thin if the dimensionless parameter is $\lambda = ha^{-1} \ll 1$, where $2a$ is the loading section of the strip.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 133-138, January-February, 1985. Original article submitted May 19, 1983.

 0021-8944/85/2601-0122$09.50